

To Constrain the Parameter Space of the ICS Model

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Abstract

The pulse profile of pulsar gives geometric information about pulsar's radiation model. After investigating the pulse profiles of PSR B1642-03 and PSR B0950+08, we calculate the ratios of beam width and the emission height between different frequencies. We find that the ratios are almost constants as inclination angle α changes from 0° to 90° . The ratios can be used to test pulsar's radiation model. In particular it can well constrain the parameter space of the inverse Compton scattering (ICS) model. These constrained parameters indicate some physical implication for the ICS model.

Key words: dense matter, pulsars: general, stars: neutron

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1 Introduction

Radhakrishnan and Cooke (1969) proposed the rotation vector model (RVM) to explain the 'S'-shape polarization position angle curve of radio pulsar. Basing on his model and the relation between beam radius ρ and the period P , $\rho \sim P^{-1/3}$, Lyne and Manchester (1988, hereafter LM88) calculated the value of inclination angle α for over 200 pulsars. Applying another $\rho - P$ relation: $\rho \sim P^{-1/2}$ and RVM, Rankin (1993) gave the values of α for 150 pulsars.

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However, the exact value of α can not be well determined for the pulsars with narrow pulse profiles, e.g. PSR 0525+21 (Gould & Lyne 1998). In this way, to determine the physical or geometrical parameters, which is sensitive to α , becomes a remarkable problem. However we find that the ratio between the beam widths at given two frequencies are almost a constant respective to the changing of α , and so do the ratio between the height of radiation location at two frequencies. Thus we can use this approximately unaltered ratio to test the radiation models. In this paper, we use the results to constraint the parameter space of inverse Compton scattering (ICS) model (Qiao, 1988) and give the physical implication.

2 The Observation and the Calculation

Kramer (1994) studied the pulse profiles of PSR B1642-03 and PSR B0950+08 and gave the pulse widths at three different frequencies. For PSR B0950+08, the pulse width increases firstly and then decreases with the increasing of frequency. But for PSR B1642-03, it only increases while the increasing of frequency. LM88 gave the maximum slope κ of the position-angle curve.

The κ is given below in RVM.

$$(d\psi/d\phi)_m = \sin \alpha / \sin \beta \equiv \kappa, \quad (1)$$

where ψ and ϕ are the polarization position angle and the phase, β is the impact angle, that is, the minimum angle between line of sight and the magnetic axis. LM88 shows that $\kappa = 50$ and 1.4 for PSR B1642-03 and PSR B0950+08 respectively, which are used to calculate the β in this paper. Next we will show that the ratio between the beam widths and the heights of radiation location at different frequencies is nearly a invariant respective to α .

The beam width θ_μ can be obtained from the observed pulse width $\Delta\phi$ by

$$\sin^2(\theta_\mu/2) = \sin^2(\Delta\phi/2) \sin \alpha \sin(\alpha + \beta) + \sin^2(\beta/2). \quad (2)$$

Now we calculate the height of radiation location. If the emission in different frequency comes from different magnetic field, the phase φ of a field line in the magnetic frame system as shown in Fig.1 can be obtained from $\cos(\alpha + \beta) = \cos \alpha \cos \theta_\mu - \sin \alpha \sin \theta_\mu \cos \varphi$.

The maximum radius of a given magnetic field lines, i.e. the furthest distance

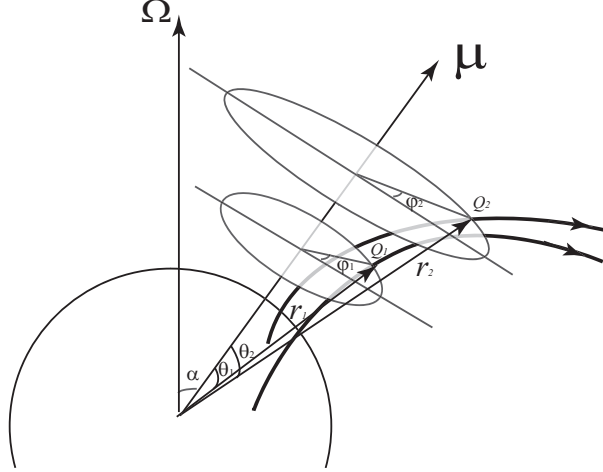


Fig. 1. Sketch of the magnetic field line, emission beams, and the emission beams (two ellipses) in two frequencies: f_1 and f_2 . Q_1 and Q_2 are two emission points. r_1 , θ_1 , φ_1 and r_2 , θ_2 , φ_2 are the emission heights, polar angles, and phases of Q_1 and Q_2 , respectively.

from the magnetic field line to the neutron star, is

$$Re = \frac{R_{LC}}{\sin^2 \theta_M \sqrt{1 - (\cos \alpha \cos \theta_M - \sin \alpha \sin \theta_M \cos \varphi)^2}}, \quad (3)$$

where R_{LC} is the radius of light cylinder and θ_M is the angle between the radius and the rotation axis when the projection of radius in the vertical plane to the rotation axis is equal to R_{LC} . Then the emission height r are given by the equation of magnetic field: $r = Re \sin^2 \theta$, θ is the polar angle at point Q as shown in Fig.1.

From the three observed pulse widths at three frequencies from Kramer (1994) and used the κ from LM88, the three relations of $\theta_{\mu 2}/\theta_{\mu 1}$ and r_2/r_1 as the function of α for PSR B1642-03 and PSR B0950+08 are calculated by above method and are shown in Fig.2 and Fig.3.

The error comes from the $\Delta\phi$ (Kramer 1994) and $\Delta\kappa$. (Here, we suppose $\Delta\kappa \approx 0.1\kappa$.)

From above two figures, we can see that the ratios are almost constants when α change from 0° to 90° . This result can also be found in the Table 1, which shows the variance $\Delta\eta$ to the mean $\bar{\eta}$. Here, $\eta_1 = \theta_{\mu 2}/\theta_{\mu 1}$, $\eta_2 = r_2/r_1$, $\Delta\eta = \eta_{\max} - \eta_{\min}$. It can be seen that the variance is rather small comparable to the mean value. Thus the ration $\theta_{\mu 2}/\theta_{\mu 1}$ and r_2/r_1 are insensitive to α at all.

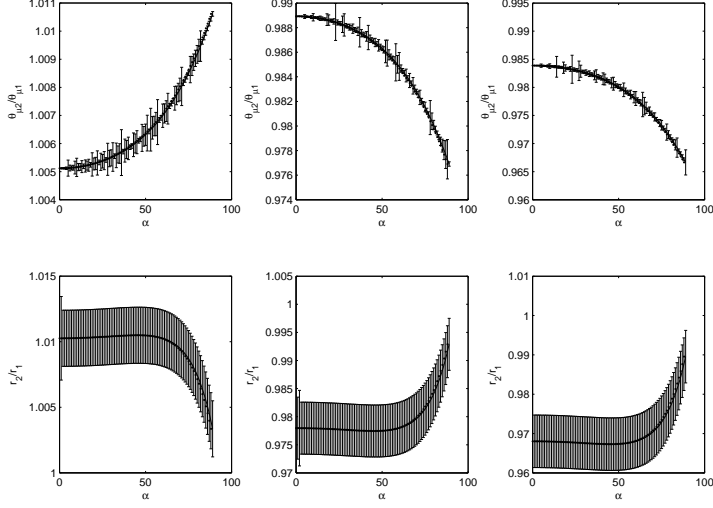


Fig. 2. $\theta_{\mu 2}/\theta_{\mu 1}$ and r_2/r_1 as the function of α for PSR B0950+08. The left, middle, and right panels are the ratios of 4.75GHz to 1.43GHz, 10.55GHz to 1.43GHz, and 10.55GHz to 4.75GHz. The error bars indicate the error passing from the errors of $\Delta\phi$ and κ .

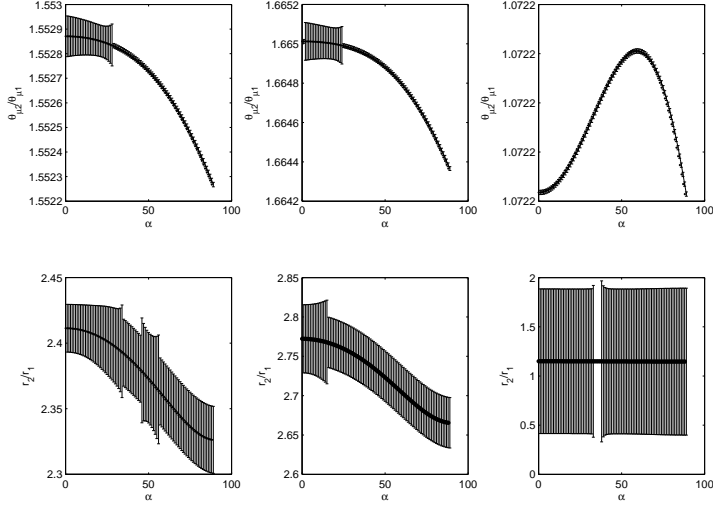


Fig. 3. $\theta_{\mu 2}/\theta_{\mu 1}$ and r_2/r_1 as the function of α for PSR B1642-03. The left, middle, and right panels are the ratios of 4.75GHz to 1.42GHz, 10.55GHz to 1.42GHz, and 10.55GHz to 4.75GHz. The error bars indicate the error passing from the errors of $\Delta\phi$ and κ .

3 The Parameter Space of ICS model

The ICS model proposed by Qiao (1988) is model including the process of a low frequency wave with angular frequency ω_0 produced in sparking and

	PSR B0950+08			PSR B1642-03		
ratio	η_a	η_b	η_c	η'_a	η'_b	η'_c
$\Delta\eta_1/\bar{\eta}_1$	5.5×10^{-3}	1.2×10^{-2}	1.8×10^{-2}	3.9×10^{-4}	3.9×10^{-4}	1.3×10^{-5}
$\Delta\eta_2/\bar{\eta}_2$	1.5×10^{-2}	1.6×10^{-2}	2.3×10^{-2}	3.8×10^{-4}	3.9×10^{-4}	3.3×10^{-3}

Table 1

The ratio of the variance $\Delta\eta$ to $\bar{\eta}$ for PSR B0950+08 and PSR B1642-03. The η_a , η_b , and η_c are the ratios of 1420 MHz to 4750 MHz, 1420 MHz to 10.55 GHz, and 4750 MHz to 10.550 GHz of PSR B0950+08, respectively. The η'_a , η'_b , and η'_c are the ratios of 1430 MHz to 4750 MHz, 1430 MHz to 10.55 GHz, and 4750 MHz to 10.55 GHz of PSR B1642-03, respectively.

scattered by high frequency secondary particles with Lorentz factor γ . Thus the observed angular frequency is

$$\omega' \simeq 2\gamma^2\omega_0(1 - \beta_0 \cos \theta_i), \quad (4)$$

where $\beta_0 = v/c \simeq 1$ as the secondary plasma moving with relativistic velocity, θ_i is the angle between the direction of motion of low energy photon and high energy particle as described by Qiao & Lin(1998).

Applying observation and the calculations in Sect.2, we constrain the parameter space of ICS model as follows. Firstly, we assume that all the emission come from the last open field line and the Lorentz factor γ is a constant for a observed frequency ¹. Secondly, because one sparking process has time scale about $10^5 - 10^6$ s, the range of ω_0 is in $10^5 - 10^6$ s⁻¹. The secondary particles have the energy with $\gamma \sim 10^2 - 10^4$. Thirdly, according to the assumption of Qiao & Lin (1998) and Eq.4, we calculate the theoretical ratios of beam width and emission height as function of α . Then we compared the simulated ratio from ICS model with the ratio measured from observation by above method to constrain the range of parameters ω_0 and γ . The results of PSR B0950+08 and PSR B1642-03 are shown in Fig.4 and Fig.5, respectively.

From the figures, we can find: 1). The values of ω_0 and γ can be fixed into a parameter space, $\omega_0 \in [10^5, 2 \times 10^5]\text{s}^{-1}$ and $\gamma \in [10^3 - 2.5 \times 10^3]$. 2). The constant γ ICS model works better at high frequencies, this may indicate that the Lorentz factor varies slower at the higher location than at lower location.

¹ The ICS model in Qiao (1988) does not assume a constant γ , here we assume a constant γ for simplicity.

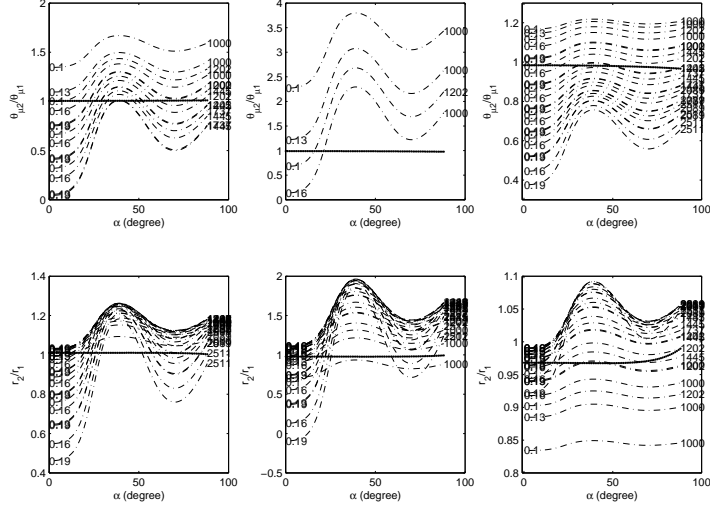


Fig. 4. Simulated and observed ratios $\theta_{\mu 2}/\theta_{\mu 1}$ and r_2/r_1 as the function of α for PSR B0950+08. The left, middle, and right panels are the same ratios as in Fig.2. The black dot line is the observed result from the pulse width. The dash dot lines are the theory results from the ICS model. Each line has different values of ω_0 and γ , which are labeled in the left and the right of the line, respectively.

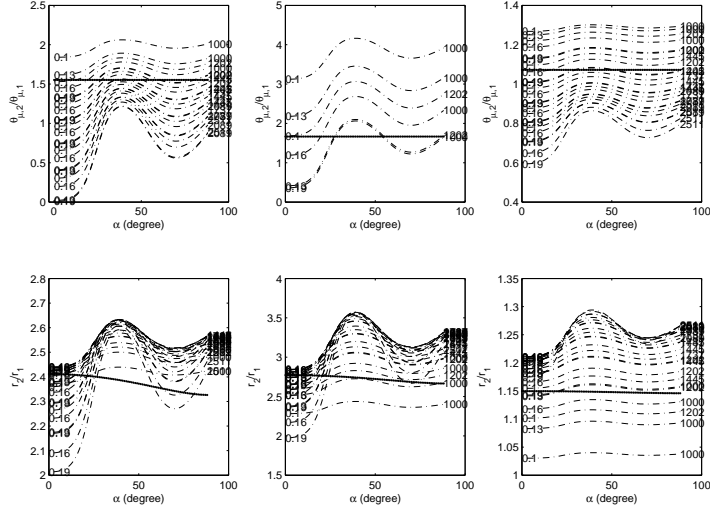


Fig. 5. Theory and observed ratios $\theta_{\mu 2}/\theta_{\mu 1}$ and r_2/r_1 as the function of α for PSR B1642-03. All the denotations are the same as those of Fig.4.

4 Conclusion and Discussion

Using the observed pulse widths at different frequencies, we show that the ratios of beam width and emission height between different frequencies for PSR B0950+08 and PSR B1642-03 are insensitive to inclination angle α . A

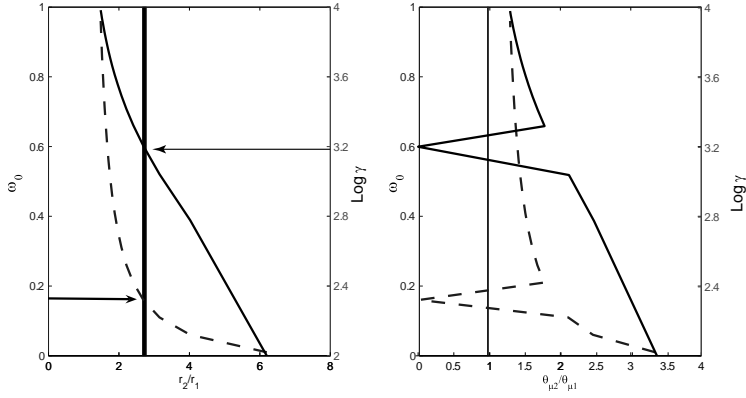


Fig. 6. ω_0 and γ as a function of r_2/r_1 for PSR B1642-03 (left panel) and ω_0 and γ as a function of $\theta_{\mu 2}/\theta_{\mu 1}$ for PSR B0950+08 (right panel). The dash and the solid curves denote the relations of ω_0 vs r_2/r_1 and γ vs r_2/r_1 . Middle black bar is the observed range. Two arrow marks give the exact values of ω_0 and γ . Left panel gives the exact values for this pulsar: $\omega_0 = 0.15$ and $\gamma = 2.5 \times 10^3$, which is consistent with the result shown in Fig.5. However, on the most of the frequencies, ω_0 and γ as the function of r_2/r_1 or of $\theta_{\mu 2}/\theta_{\mu 1}$ are not monotonic functions as shown in right panel. Therefore, it is hard to give the exact value but only the parameter space.

constant ratio indicates that α does not remarkably affect these ratios. These constants derived from observation can constrain the parameter space of ICS model. The comparison of observation and simulated results show that the ICS model might work better in high frequencies than that in low ones.

In the work, we only give a roughly parameter space (see Fig.6 for details) but not give the exact values for the parameters. We hope that this work can give hints to the future work to determine the exact value of parameters about pulsar's radiation geometry and to test more pulsar's radiation model.

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